Exercise 42

Find the limit or show that it does not exist.

$$\lim_{x\to\infty}[\ln(2+x)-\ln(1+x)]$$

Solution

Use the property of logarithms that allows a difference to be written as a quotient. Then multiply the numerator and denominator by the reciprocal of the highest power of x in the denominator.

$$\begin{aligned}
\lim_{x\to\infty}[\ln(2+x)-\ln(1+x)] &= \lim_{x\to\infty}\ln\frac{2+x}{1+x} \\
&= \lim_{x\to\infty}\ln\frac{2+x}{1+x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} \\
&= \lim_{x\to\infty}\ln\frac{(2+x)\frac{1}{x}}{(1+x)\frac{1}{x}} \\
&= \lim_{x\to\infty}\ln\frac{\frac{2}{x}+1}{\frac{1}{x}+1} \\
&= \ln\frac{\lim\limits_{x\to\infty}\left(\dfrac{2}{x}+1\right)}{\lim\limits_{x\to\infty}\left(\dfrac{1}{x}+1\right)} \\
&= \ln\frac{\lim\limits_{x\to\infty}\dfrac{2}{x}+\lim\limits_{x\to\infty}1}{\lim\limits_{x\to\infty}\dfrac{1}{x}+\lim\limits_{x\to\infty}1} \\
&= \ln\frac{0+1}{0+1} \\
&= \ln 1 \\
&= 0
\end{aligned}$$